MEBA Engineering School

BASIC MATHEMATICS FOR ELECTRICITY

(Revised: January 2002)

This book belongs to: ____

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Preface

Most of this material is extracted from Mathematics for Electricity and Electronics, Arthur D. Kramer, 1st ed., Delmar Publishers, Inc., ISBN 0-8273-5804-0. (A second edition has been since released, but, in our opinion, is not a material improvement over what we sought from the first edition.) We have slightly modified Mr. Kramer's material for our needs in the MEBA Electricity Course and the Electrical Troubleshooting Course.

This text is available from the MEBA Bookstore. Call Kim Cooper at 410+822-9600, ext. 321 for availability and current price. This material (not the entire Kramer text) is also available on a CD for your use.

We review Chapters 1 through 5 at the beginning of the Electricity Course. We also review Section 1.4, The Concept of Powers of 10, at the beginning of the Electrical Troubleshooting Course. This often provides a degree of compensation for the variable mathematical backgrounds of Members. Another problem is that the Member's background in mathematics may be a number of years past, hence largely forgotten as a result of disuse.

Mr. Kramer begins each topic area with a generalized mathematical definition of the topic. The Member will however, will find the examples, which follow the definition to be quite inclusive and helpful.

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BASIC MATHEMATICS
FOR ELECTRICITY

The Learning Objectives of this Program are:

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1. To review *Powers of Numbers* as needed for the Electrical Troubleshooting Course (#) and the Electricity Course. (#)

2. To review *Basic Algebra* as needed for the Electricity Course. (#, ##)

3. To review *Basic Trigonometry* as needed for the Electricity Course. (#, ##)

4. To present a simplified approach to the solution of mathematical *word problems*. (##)

5. To obtain a very basic understanding of the concepts upon which *Differential and Integral Calculus* are based. (##)

6. To obtain an understanding of logarithms and their use. (for Shipboard Electrical Systems Course)

* Indicates that a Quiz follows.
** Indicates for the Electricity Course only.

Acknowledgements:

The bulk of the following material is taken from *Mathematics for Electricity and Electronics,* Arthur D. Kramer, Delmar Publishers Inc., ISBN 0-8273-5804-0.

This book is available from the Bookstore at the Calhoon School.

Some is also taken from the 1995 issue of *the ARRL Handbook for Radio Amateurs* (!), Robert Schetgen KU7G, editor, published by The American Radio Relay League, Newington, CT 06111, ISBN 0-87259-172-7.

Chapter 1

Powers of Numbers

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Powers of Numbers

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Learning Objectives

In this Chapter you will learn:

> How to raise a number to a power.

> The rules of working with powers of ten.

> How to find the roots of powers of ten.

> How to raise powers of ten to a positive number.

> How to find the square root of a number.

> How to find the cube root of a number.

> How to simplify expressions with radical signs.

> How to find powers and roots on the calculator.

> How to utilize scientific or engineering notation

> How to handle work problems.

1.1 What is a Power?

A *power* or *exponent* defines repeated multiplication.

Generalized:

$$x^n = (x)(x)(x).....(x)$$

which is *(x)* times itself n times
(where n is a positive whole number)

examples:

(1) $$2^6 = (2)(2)(2)(2)(2)(2) = 64$$

(2) $$6^3 = (6)(6)(6) = 216$$

There are specific keys for accomplishing this on your pocket calculator without requiring tedious repeated multiplication.

Consider these other examples:

(3) $$\left(\frac{2}{5}\right)^4 = \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right) = \frac{16}{625}$$

or: $$\left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4} = \frac{16}{625}$$

(4) $(0.01)^2 = (0.01)\ (0.01) = 0.0001$

Since there are *two* decimal places in 0.01, squaring the number (which is multiplying it by itself) will result in *four* decimal places.

This is much easier done on the pocket calculator using the square key because the calculator memory keeps track of the number of zeros for you.

There are ***Order of Operations Rules*** in mathematics.

Of interest to us here is the Order of Operations Rule which dictates *raising to a power (or taking a root)* ***first,*** (before multiplication or division).

for example:

(5) $$\frac{(2)^5(7)^2}{4^3} = \frac{(32)(49)}{64} = \frac{49}{2} = 24.5$$

(6) $$\left(\frac{3}{2}\right)^2 - 6\left(\frac{1}{2}\right)^3 = \frac{9}{4} - \frac{6}{8} = \frac{9}{4} - \frac{3}{4} = \frac{6}{4} = \frac{3}{2} = 1.5$$

(7) $$\frac{(0.1)^2 + (0.3)^3}{0.5}$$

Raise to the powers *first*, do the indicated addition in the numerator, and finally do the division.

$$\frac{(0.1)^2 + (0.3)^3}{0.5} = \frac{0.01 + 0.027}{0.5} = \frac{0.037}{0.5} = 0.074$$

1.2 Roots

The *square root* of a given number is the number, which, when squared, exactly equals the given number.

The square root operation is indicated by the root or radical sign (√).

examples:

(1) $\sqrt{64}$ = 8 because $8^2 = 64$

(2) $\sqrt{0.16} = 0.4$ because $(0.4)^2 = 0.16$

other examples:

(3) $\sqrt{\frac{16}{49}} = \frac{\sqrt{16}}{\sqrt{49}} = \frac{4}{7}$

(4) $\sqrt{1.44} = 1.2$ (this is more readily done on a pocket calculator)

Numbers which are not perfect squares, such as 2, 3, 5, and 7, etc., have square roots which are *infinite decimals*, and it is necessary to <u>round off</u> the result:

examples:

(5) $\sqrt{2} = 1.414213562\ldots\ldots \cong 1.414$

(6) $\sqrt{7} = 2.645751311\ldots\ldots \cong 2.646$

The symbol ($\cong$) means *is approximately equal to.* That is, $\sqrt{2}$ is approximately 1.414, and $\sqrt{7}$ is approximately 2.646.

Three decimal places, i.e., three digits to the right of the decimal sign, is typical in most work.

For any positive number x, the definition of the square and square root are inverse operations, and tend to "cancel each other out." When a square root and a square appear together, the radical and the square signs cancel:

$$\sqrt{x^2} = \left(\sqrt{x}\right)^2 = x$$

for a numerical example:

(7) $\sqrt{9^2} = 9$ and $\left(\sqrt{9}\right)^2 = 9$

Two basic rules of calculation for the square roots of two positive numbers, x and y, are:

$$\sqrt{xy} = \left(\sqrt{x}\right)\left(\sqrt{y}\right)$$

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

These rules help to simplify radical expressions. Products or quotients under the radical sign can be separated into products or quotients of separate radicals, and vice versa.

example:

(8) $\sqrt{12} = \sqrt{(4)(3)} = \left(\sqrt{4}\right)\left(\sqrt{3}\right) = 2\sqrt{3}$

If the number under the radical (or root) sign contains a factor which is a perfect square, such as the factor 4 above, it can be simplified by separating the perfect square.

Note that the number in front of the radical means it is multiplied by the radical:

$$2\sqrt{3} = 2x\sqrt{3}$$

These rules also show that you can multiply under the radical:

$$\left(\sqrt{0.2}\right)\left(\sqrt{1.8}\right) = \sqrt{(0.2)(1.8)} = \sqrt{0.36} = 0.6$$

Note that the above procedure applies only to *like radicals*, such as square roots.

specific examples:

(9) find: $\sqrt{75}$

The radical can be simplified by looking for a perfect square which is a factor of the number 75. Clearly, this factor is 25, therefore:

$$\sqrt{75} = \left(\sqrt{25}\right)\left(\sqrt{3}\right) = 5\sqrt{3}$$

examples:

(10) find: $\sqrt{\frac{3}{100}} = \frac{\sqrt{3}}{\sqrt{100}} = \frac{\sqrt{3}}{10} = 0.1\sqrt{3}$

(11) find: $\frac{\sqrt{28}}{\sqrt{7}} = \sqrt{\frac{28}{7}} = \sqrt{4} = 2$

(12) find: $\sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{\sqrt{9}} = \frac{\left(\sqrt{4}\right)\sqrt{2}}{3} = \frac{2\sqrt{2}}{3} = \frac{2}{3}\sqrt{2}$

1.3 Cube Roots

Similar to the square root, the cube root is the inverse of raising to the *third power*.

A cube root is written using the radical sign with the *index 3*.

$$\sqrt[3]{}$$

The index of the *square root* was not shown; it is, therefore, *understood to be 2*:

$$\sqrt{} \equiv \sqrt[2]{}$$

where the symbol ($\equiv$) means *identical to*.

Cube root, or third root, examples are:

(1) $\sqrt[3]{1{,}000} = 10$ because $10^3 = 1{,}000$

(2) $\sqrt[3]{0.064} = 0.4$ because $(0.4)^3 = 0.064$

The definition of a cube root of a number x is similar to that for a square root:

$$\sqrt[3]{x^3} = \left(\sqrt[3]{x}\right)^3 = x$$

Since raising to the third power is the inverse of taking a cube root, the two operations cancel each other.

for example:

(3) $$\sqrt[3]{8^3} = 8$$

(4) $$\left(\sqrt[3]{8}\right)^3 = 8$$

The cube root operation can also be done with the pocket calculator, but not quite as simply as the square root.

Using the **2nd** and ^{x}**Õy** keys you can take *any* root of a number. Try this.

<u>Practice Problems</u>:

> ***Kramer, pages 46 & 47, problems 1 - 44.***

1.4 The Concept of Powers of 10:

Our number system is the *decimal system*. It is based on 10 and powers of 10.

When we write the number 546 we really mean:

$$546 = (5 \times 10^2) + (4 \times 10^1) + (6 \times 10^0)$$

where: $10^2 = 100$
$10^1 = 10$
$10^0 = 1$

$$546 = (5 \times 100) + (4 \times 10) + (6 \times 1)$$
$$500 + 40 + 6$$

This gives different numbers different weights, *depending upon their positions*.

Generalizing about powers of 10, the *base*, which in our number system is 10, is simply *multiplied by itself* a number of times indicated by the *exponent* (or power).

Powers of 10 may be thought of as a shorthand notation system which has advantages in writing very large or very small numbers.

Since a large number of zeros are difficult to handle, it's a lot easier to write:

10^{7} than $10{,}000{,}000$

or: 10^{-6} than $\dfrac{1}{1{,}000{,}000}$

Ah!; we also have *negative powers of ten!* The negative power is simply read as *one over*, i.e.:

$$10^{-1} = \frac{1}{10} \qquad \text{etc.}$$

This table shows several powers of 10 which are common to electricity and electronics. The commonly used prefixes are also included.

Prefix	***Power of Ten***	***Decimal***	***Fraction***
mega (M)	10^{6}	1,000,000.0	1,000,000
kilo (K)	10^{3}	1,000.0	1,000
unity	10^{0}	1.0	1
milli (m)	10^{-3}	0.001	$\frac{1}{1{,}000}$
micro (μ)	10^{-6}	0.000001	$\frac{1}{1{,}000{,}000}$
pico (p)	10^{-12}	0.000000000001	$\frac{1}{1{,}000{,}000{,}000{,}000}$

Note that for every *increase* of one in the power of 10, the decimal point moves one place to the *right*.

For 0, 1, 2, 3, etc., the power is equal to the number of zeros.

$10^0 = 1$

$10^3 = 1,000$

$10^6 = 1,000,000$

For every *decrease* of in the power of 10, the decimal point moves one place to the *left*. Therefore, negative powers of 10 are decimals between 0 and 1.

Note that for *positive* powers of 10 we do not include the (+) symbol; it is *understood*.

For any positive number n:

$10^{-n} = 1/10^n$

1.5 Multiplication and Division with Powers

When multiplying with powers of 10, *add* the exponents *algebraically*.

examples:

(1) $10^6 x 10^{-2} = 10^{6-2} = 10^4$

(2) $10^{-6} x 10^4 = 10^{(-6+4)} = 10^{-2}$

(3) $10^3 x 10^{-9} x 10^0 = 10^{+3-9+0} = 10^{-6}$

note: $10^0 \equiv 1$ (multiplying by one does not change anything)

When dividing with powers of 10, subtract the exponents algebraically. Change the sign of the power in the divisor, then multiply by adding the exponents algebraically.

example:

(4) $\frac{10^2}{10^6}$

One approach is to subtract these exponents by change 6 to -6 and add the exponents algebraically.

examples:

(5) $10^{2}x10^{-6} = 10^{-4}$

(6) $\dfrac{10^{-3}}{10^{3}} = 10^{-3}x10^{-3} = 10^{-6}$

A very large or very small number is often most conveniently expressed in terms of a number *times* a power of 10.

examples:

(7) $7{,}000{,}000 = 7x1{,}000{,}000 = 7x10^{6}$

(8) $0.004 = 4x0.001 = 4x10^{-3}$

When multiplying or dividing numbers with powers of 10, calculate each part separately.

1. Multiply or divide the numbers to obtain the *number part* of the answer.

2. Apply the rules for multiplying or dividing with powers of 10 to obtain the applicable power of 10.

examples:

(9) $\left(2x10^{2}\right)\left(6x10^{3}\right) = (2)(6)x10^{2+3} = 12x10^{5}$

(10) $\dfrac{8x10^{-3}}{4x10^{-1}} = \dfrac{8}{4}x\left(10^{-3}\right)\left(10^{1}\right) = 2x10^{-2}$

(11) $$\frac{(4x10^{-6})(6x10^{3})}{(8x10^{-3})} = \frac{(4)(6)x10^{(-6+3)}}{8x10^{-3}} = \frac{24x10^{-3}}{8x10^{-3}} = 3x10^{(-3+3)} = 3$$

1.6 Addition and Subtraction with Powers

Numbers times powers of 10 cannot be added or subtracted in power of 10 format *unless the powers are exactly the same.*

If the powers are exactly the same, add (or subtract) the numbers only. Do not change the power of 10.

example:

(1) $$(5x10^{6}) + (7x10^{6}) = (7+5)x10^{6} = 12x10^{6}$$

When the powers are *different,* the numbers must be changed to ordinary notation, or to like powers, before adding or subtracting.

example:

(2) $$(2x10^{-3}) - (9x10^{-4})$$

change to ordinary notation -

$$(2x0.001)-(9x0.0001)=0.0020-0.0009=0.0011=1.1x10^{-3}$$

or change to like powers of 10 -

$$\left(2x10^{-3}\right)-\left(9x10^{-4}\right)=\left(2x10^{-3}\right)-\left(0.9x10^{-3}\right)=1.1x10^{-3}$$

note: $4x10^{5}=0.4x10^{6}$ and $7x10^{-8}=70x10^{-9}$

1.7 Raising to Powers and Taking Roots

To raise a number times a power of 10 to a higher power, raise the number to the higher power and *multiply* the powers.

example:

(1) $\left(4x10^{3}\right)^{2}=4^{2}\,x10^{3x2}=16x10^{6}$

To take a root of a number times a power of 10, take the root of the number and divide the index of the root into the power of 10.

example:

(2) $\sqrt[3]{\left(27x10^{9}\right)}=\sqrt[3]{27}x10^{9/3}=3x10^{3}$

The index of the root is the small number to the upper left of the radical sign. For a square root, the index is 2 and it is understood without being stated.

example:

(3) $$\frac{\sqrt{9x10^{6}}}{\left(2x10^{3}\right)^{3}\left(5x10^{-3}\right)}$$

Apply the Order of Operations and do the powers and roots first.

Find the square root of the numerator and raise the number to the power in the denominator:

$$\frac{\sqrt{9x10^{6}}}{\left(2x10^{3}\right)^{3}\left(5x10^{-3}\right)}=\frac{\sqrt{9}x10^{6/2}}{\left(2^{3}\,x10^{3x3}\right)\left(5x10^{-3}\right)}=\frac{3x10^{3}}{\left(8x10^{9}\right)\left(5x10^{-3}\right)}$$

Then multiply the numbers in the denominator and divide:

$$\frac{3x10^{3}}{\left(8x10^{9}\right)\left(5x10^{-3}\right)}=\frac{3x10^{3}}{40x10^{6}}=\frac{3}{40}x10^{(3-6)}=0.075x10^{-3}=75x10^{-6}$$

Note that the above answer may be written more than one way with a power of 10 by appropriately moving the decimal point.

<u>Practice Problems</u>:

Kramer, pages 54 & 55, problems 1 - 50.

1.8 Scientific and Engineering Notation

Consider the following problem:

The brightest star in the sky, Sirius, is 8.7 light-years away. This means it takes 8.7 years for light to travel from Sirius to the earth.

If light travels at 186,000 miles/second, how far away is Sirius?
Express the answer to *three significant digits*.

Answer:

(186,000 mil/sec)(60 sec/min)(60 min/hr)(24 hr/day)(365 days/yr)(8.7 yr)

= 51,000,000,000,000 miles, or 51 trillion miles

Similar very large or very small numbers are often found in scientific work, particularly in electronics. These numbers are expressed in powers of 10 in *scientific or engineering notation*.

In *scientific notation*:

$51,000,000,000,000 = 5.1 \times 10^{13}$ miles

Engineering notation, however, employs powers of 10 which are *divisible by three.*

In *engineering notation* the above distance would be expressed as:

51×10^{12} miles

Powers of 10 which are divisible by 3 correspond to units used in electricity and electronics, as well as in the metric system.

examples:

(1) 10^3 is kilo (as in kilo ohms)

(2) 10^{-3} is milli (as in milliamperes)

(3) 10^{-6} is micro (as in microfarads)

(4) 10^6 is mega, etc. (as in megohms)

***Practice Problems*:**

Kramer, page 61, problems 1 - 30.

1.9 The Difficulty of Word Problems

(This is a bonus section; no extra charge!)

The thought of word problems in mathematics makes many brave persons freeze. The clue to handling word problems is in your approach to them. Let's consider problem #32 from page 61 of *Kramer*:

> Parker Brothers, Inc., manufacturer of the board game *Monopoly*, printed $18,500,000,000,000 of toy money in 1990 for all its games, which is more than all the real money in circulation in the world. If all this "money" were distributed equally among the world's population, estimated at 5.3 billion in 1990, how much would each person get?

The prime task in a work problem is to separate “the wheat from the chaff.”

1. Go through the text of the problem and pull out the numeric *facts* given. State them as briefly as possible, consistent with clarity:

printed $18,500,000,000,000 of toy money

~~printed the toy money in 1990~~

world population 5,300,000,000 ~~in 1990~~

2. What is the *question?* State it as briefly as possible.

how much money would each person get

3. *Review the facts;* are they all needed to solve the problem, or are some irrelevant? Cross out any that are irrelevant:

1990 is relevant only to the extend that the amount printed and the population are on a mutually consistent timetable. It will not play a
part in problem solving, therefore is crossed out of the relevant facts statements.

4. Having separated the “wheat from the chaff,” *solve the problem.*

$$\frac{18{,}500{,}000{,}000{,}000 \text{ dollars}}{5{,}300{,}000{,}000 \text{ people}}$$

3,490.57 dollars per person

1.10 Acknowledgment:

The information contained on the preceding pages, excepting the final section on *Word Problem Solving*, has been plagiarized (with minor modifications) from Chapter 2 of:

> *Mathematics for Electricity and Electronics*, Arthur D. Kramer
> Delmar Publishers Inc., 1995, ISBN 0-8273-5804-0

Questions, as well as Applications to Electronics, are found as follows:

Powers and Roots:
Powers, Square Roots and *Cube Roots*, pages 46 - 48

Powers of 10:
Introduction, *Multiplication and Division*, *Addition and Subtraction*, *Raising to Powers and Taking Roots*, pages 55 & 56

Scientific and Engineering Notation, pages 61 - 63

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Chapter 2

Basic Algebra

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Basic Algebra

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Learning Objectives

In this Chapter you will learn:

> The meaning of signed numbers.

> How to add, subtract, multiply and divide signed numbers.

> How to identify an algebraic expression.

> How to combine algebraic terms.

> The basic rules for working with exponents.

> The meaning of zero and negative exponents.

> How to work with Ohm’s Law and the Power Formula.

2.1 Signed Numbers

One of the basic features of algebra is the use of *positive and negative signed numbers.*

The signed numbers and zero can be represented as a *number line*, where the numbers increase *from left to right*:

..... -3, -2, -1, 0, +1, +2, +3, ...

Notice that -2 is *mathematically a larger* number than -3 because the numbers *increase* from left to right.

The sign is also used to indicate *direction* in electricity and electronics. For example, a voltage of zero usually indicates the ground reference potential, with *positive* meaning a voltage above ground and *negative* a voltage below ground.

A voltage of +5 V has the same *magnitude* or *absolute value* as a voltage of -5 V, but with a different reference point.

For current, positive indicates current in one direction, and negative indicates current in the opposite direction.

A current of +2 ma has the same magnitude as a current of -2 ma, but it flows in the opposite direction.

Positive and negative numbers are also used for angles, where positive represents counterclockwise rotation and negative represents clockwise rotation.

Positive and negative numbers are used for powers of 10, as we have seen.

Positive and negative whole numbers, including zero, are called *integers*.

The *absolute value* of an integer is the value of the number *without the sign*, and it represents the *magnitude* of the number.

Absolute values are equal to the positive value and are indicated by *vertical lines*:

examples:

(1) $|-6| = 6$

(2) $|+9| = 9$

(3) $|-415| = 415$

The rules for adding, subtracting, multiplying and dividing signed numbers are:

1. Adding Like Signs -

To add two signed numbers with like signs, add the numbers and apply the common sign.

examples:

(4) (-8) + (-7) = -8 - 7 = -15

(5) 6 + (+2) = +8 or simply 8

Note that in the absence of a sign, the number is understood to be positive, i.e.:

(6) 5 means +5

2. Adding Unlike Signs -

To add two signed numbers with unlike signs, subtract the smaller absolute value from the larger absolute value.
Then apply the sign of the larger absolute value.

examples:

(7) (-9) + (+6) = -9 + 6 = -3

(8) 13 + (-5) = +13 - 5 = +8

To add *several* signed numbers, add the like signs first, then take the difference of the positive and negative numbers:

examples:

(9) $6.6 + (-2.3) + (-8.8) + 1.2 = (6.6 + 1.2) - (2.3 + 8.8)$

$$= 7.8 - 11.1 = -3.3$$

3. Subtracting Signed Numbers -

To subtract two signed numbers, change the sign of the number that follows the negative sign.
Then add the numbers applying the rules above.

examples:

(10) $(10) - (-3) = 10 + (+3) = 10 + 3 = 13$

(11) $(-14) - (+6) = -14 + (-6) = -14 -6 = -20$

(12) $(12) - (7) = 12 -7 = 5$

The minus sign just has the effect of changing the sign that follows.
A minus followed by a minus equals a plus.
A minus followed by a plus equals a minus:

example:

(13) $3.2 + (- 4.1) - (5.5) - (1.2)$

$3.2 - 5.5 - 4.1 - 1.2 = 3.2 – 10.8 = -7.6$

4. Multiplying or Dividing Signed Numbers -

To multiply or divide two signed numbers, multiply or divide the absolute values.
Then make the result positive if the signs are the same, negative if the signs are different.

examples:

(14) $(-5)(-3) = +15$

(15) $(3)(-5)(2) = -15(2) = -30$

(16) $(5)(-3) \div 15 = -15 \div 15 = -1$

Multiplication of numbers in algebra is indicated by parentheses or a dot: (•):

example:

(17) $(-5)(3) = -5 \bullet 3$

The multiplication sign (x) is better avoided in algebra because it can be confused with the *letter x,* often used to designate an unknown value.

The *Order of Operations in Arithmetic* also applies in algebra:

1. Perform operations in parentheses.

2. Do multiplication or division.

3. Do addition or subtraction.

examples:

(18) $$-0.5(-7+3)+\frac{8(-3)}{6}$$

Perform the operations in parentheses first,
then multiply or divide,
finally, add or subtract

$$-0.5(-4)-\frac{24}{6}=2+\frac{-24}{6}=2+(-4)=-2$$

(19) $$\frac{-8(3)-4(2)}{2+3(-1)}=\frac{-24-8}{2-3}=\frac{-32}{-1}=32$$

2.2 Combining Terms

An *algebraic term* is a combination of letters, numbers, or both; joined by the operations of multiplication or division.

examples:

(1) $4x^2$

(2) $10b$

(3) $-3I^2R_1$

(4) $\frac{V_T}{R_T}$

(5) $-1.8C$

(6) $2\pi r$

Each algebraic term has two parts:

the *coefficient*, which is the constant number in front,

and the *literal part*-

Coefficient	**Literal part**
4	x^2
-3	I^2R_1
2π	r

Note that π is part of the coefficient, because it is a constant number; $2\pi = 6.28$.

The literal part consists of letters that can take on different values, and are called variables.

The subscript 1 used in the above term for R_1, means a specific value of the variable R.

R_1 is a different variable than R_2 or R_T of just R.

When the coefficient is 1, it is understood and so is not written:

examples:

(6) $\frac{V_T}{R_T} = (1)\frac{V_T}{R_T}$

(7) $-\pi d = -(1)\pi d$

Note that the minus sign represents a coefficient of -1.

Like terms are terms that have *exactly the same literal part*.

examples:

(8) $4x^2$ and $-10x^2$

(9) $-3I^2R_1$ and $5I^2R_1$

Like terms can be combined, or added together.

Unlike terms do not have the same literal part.

examples:

(10) $4x^2$ and $3x$

(11) $-3I^2R_2$ and $5I^2R_1$

$-3I^2R_2$ and $5I^2R_1$ are unlike terms, because the subscripts are different.

Unlike terms represent *different things,* and cannot be combined.

<u>Combining Like Terms</u> -

To combine like terms, add the coefficients only.
Do not change the literal part, including the exponents.

examples:

(12) $-5y + 3y = (-5 + 3)y = -2y$

(13) $6I^2R_1 - 3I^2R_1 + 2I^2R_1 = (6 - 3 + 2)I^2R_1 = (8 - 3)I^2R_1 = 5I^2R_1$

(14)
$$\begin{aligned} 5V_x + V_y - (4V_y - 3V_x) &= 5V_x + V_y - (4V_y) - (-3V_x) \\ &= 5V_x + V_y - 4V_y + 3V_x \\ &= (5 + 3)V_x + (1 - 4)V_y \\ &= 8V_x - 3V_y \end{aligned}$$

A set of parentheses inside another parentheses is often helpful for grouping.

Parentheses ( ) are used first, then brackets [ ], then braces { }.

example:

(15) $2V - [(2V - 2IR) - (V - IR)]$

Work from the inside out.
Watch negative signs in front of parentheses and brackets.

$2V - [(2V - 2IR) - (V - IR)] = 2V - [2V - 2IR - V + IR]$

Notice that the negative sign effects *all* terms within the brackets.

$= 2V - 2V + 2IR + V - IR$

Next, combine like terms.

$= V + IR$

Practice Problems:

Kramer, pages 86 & 87, problems 1 - 48.

2.3 First Degree Equations

A *first-degree* equation, or *linear* equation, is statement of equality that contains only variables to the *first power* or degree.

examples:

(1) $3t + 7 = 16$

(2) $27 = \frac{5}{9}(F - 32)$

(3) $\frac{1}{R_T} = \frac{1}{7.5} + \frac{1}{5.0}$

Note that the variables *t, F* and R_T all have an exponent of 1.

<u>To solve for the variable (or unknown)</u> -

Adding, subtracting, multiplying, or dividing each term on *both sides* of an equation by the *same quantity* does not change the *equality* of the equation. (Division by 0 is not allowed.)

examples:

(4) $3t + 7 = 16$

We need to isolate the unknown, t.
First isolate the 3t term by subtracting 7 from both sides.

$3t + 7 - 7 = 16 - 7$

Combine like terms.

$3t = 9$

Solve for t by dividing both sides by 3.

$$\frac{3t}{3} = \frac{9}{3}$$

$$t = 3$$

To check your solution, substitute t = 3 into the original equation:

3(3) + 7 = 16
9 + 7 = 16
16 = 16 ✔

(5) $$4P - 3 = 5 + 2P$$

Put all terms containing the unknown on one side of the equation and the constant terms on the other side.

First, eliminate 3 on the left and the 2P on the right.

Add 3 to both sides:

$$4P - 3 + 3 = 5 + 2P + 3$$

$$4P = 8 + 2P$$

Subtract 2P from both sides:

$$4P - 2P = 8 + 2P - 2P$$

Combine like terms, then divide both sides by 2:

$$\frac{2P}{2} = \frac{8}{2}$$

$$P = 4$$

To check the answer:

4(4) - 3 = 5 + 2(4)
16 - 3 = 5 + 8
13 = 13 ✔

(6) $$C = \frac{5}{9}(F - 32)$$

Find *F* when *C* = 27°

Substitute *C* = 27 into the formula:

$$27 = \frac{5}{9}(F - 32)$$

Eliminate fractions as soon as possible to make the solution easier. Do so by multiplying both sides by 9:

$$27(9) = \frac{5}{9}(F - 32)(9)$$

$$243 = 5(F - 32)$$

Remove the parenthesis:

$$243 = 5F - 160$$

Add 160 to each side to isolate 5*F*:

$$243 + 160 = 5F - 160 + 160$$

$$403 = 5F$$

Divide both sides by 5:

$$\frac{403}{5} = \frac{5F}{5}$$

$$\frac{403}{5} = F$$

But we are accustomed to writing it as:

$$F = \frac{403}{5} = 80.6 \text{ degrees}$$

Check out your work:

$$27 = \frac{5}{9}[(80.6) - 32)]$$

$$27 = \frac{5}{9}[48.6]$$

$27 = 27$ ✔

(7) $$\frac{5V}{2} + \frac{2V}{5} = 3(V - 3)$$

Eliminate the fractions by multiplying each term by the Least Common Denominator (LCD), which is 10:

$$\frac{5V}{2}(10) + \frac{2V}{5}(10) = 3(V - 3)(10)$$

$$5V(5) + 2V(2) = 30(V - 3)$$

Remove the parenthesis:

$$25V + 4V = 30V - 90$$

$$29V = 30V - 90$$

Collect terms by subtracting 30V from both sides:

$$29V - 30V = 30V - 90 - 30V$$

$$-V = -90$$

$$V = 90$$

To check the answer:

$$\frac{5(90)}{2} + \frac{2(90)}{5} = 3[(90) - 3]$$

$$\frac{5(90)}{2}(10) + \frac{2(90)}{5}(10) = 3[(90) - 3](10)$$

$$5(90)5 + 2(90)2 = 30[(90) - 3]$$

$$25(90) + 4(90) = 30(87)$$

$$2250 + 260 = 2610$$

$$2610 = 2610 \quad \checkmark$$

<u>Practice Prblems</u>:

Kramer, pages 131 & 132, problems 1 - 48.

1.4 Working with Ohm's Law

The use of Ohm's Law is basic to all electricity and electronics. It is usually stated as:

1. $V = IR$

where: V = voltage, in volts
I = current, in amps
R = resistance, in ohms

Ohm's law is a statement that the voltage across a resistance is equal to the product of the current flowing through the resistance and the value of the resistance.

Ohm's Law can be expressed in another way which is more useful if the voltage and the resistance are both the known quantities:

$$\frac{V}{I} = \frac{IR}{I}$$

$$\frac{V}{I} = R \quad \text{or}$$

2. $$R = \frac{V}{I}$$

The third expression of Ohm's Law is useful if the two quantities known are V and I:

$$\frac{V}{VI} = \frac{IR}{VI}$$

$$\frac{1}{I} = \frac{R}{V} \quad \text{or}$$

3. $$I = \frac{V}{R}$$

examples:

(1) Given: R = 100 ohms
V = 6 volts

Find *I:*

$$I = \frac{V}{R} \quad \text{(\#3, above)}$$

$$I = \frac{6}{100} = 6\left(10^{-2}\right) = 0.06A = 60ma$$

(2) as above, but the applied voltage is doubled:

$$I = \frac{V}{R} = \frac{12}{100} = 12\left(10^{-2}\right) = 0.12A = 120ma$$

Note that the resulting current is directly proportional to V (V is in the numerator) and inversely proportional to R (because R is in the denominator).

(3) Measured quantities are:

V = 120 volts
I = 10 amps

Find *R.*

$$R = \frac{V}{I} \quad \text{(\#2, above)}$$

$$R = \frac{120}{10} = 12\Omega$$

R is directly proportional to V, hence if V were to increase (or decrease) the resulting R would increase (or decrease) in direct proportionality.

R is also inversely proportional to I, hence if I were to increase (or decrease) the resulting R would decrease (or increase).

1.5 Electric Power

Electric power is energy per unit time, i.e., the *rate of use of electric energy.* One watt of power is equal to the energy produced every second by one volt and a current of one ampere. Thus:

$$P = VI$$

where: *P* = power, in watts

V = voltage, in volts

I = current, in amps

example:

(1) If 20 *V* produces 1.5 *A* in a circuit, what is the power?

$$P = VI = 20(1.5) = 30 \text{ watts}$$

As with Ohm's Law, the power formula can also be expressed in terms of *I* or *V:*

$$I = \frac{P}{V}$$

$$V = \frac{P}{I}$$

Ohm's law and the power formula can be combined:

$$P = VI = (IR)I = I^2 R \quad \text{and}$$

$$P = VI = \frac{V(V)}{R} = \frac{V^2}{R}$$

examples:

(2) A 12 *V* battery supplies a bulb whose resistance is 24 ohms. What is the power and the current in the circuit?
(always draw and label the circuit if it will help you in correctly visualizing the problem)

V = 12 volts

R = 24 ohms

$$P = \frac{V^2}{R} = \frac{12^2}{24} = \frac{144}{24} = 6 \text{ watts}$$

$$I = \frac{V}{R} = \frac{12}{24} = \frac{1}{2} = 0.5A = 500ma$$

(3) A resistance of 60 ohms in a DC circuit consumes 1.5 kW of power.

What voltage produces this situation?

$$P = \frac{V^2}{R}$$

$$1{,}500 = \frac{V^2}{60}$$

$$1{,}500(60) = V^2$$

$$V = \sqrt{1.5(6)10^4}$$

$$V = \sqrt{9\left(10^4\right)} = 3\left(10^2\right) = 300 \text{ volts}$$

<u>Practice Problems</u>:

Kramer, pages 133 & 134, problems starting with #49
And pages 163 & 164, problems starting with #27.

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Chapter 3

Basic Trigonometry

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Basic Trigonometry

Learning Objectives

In this Chapter you will learn:

> The basic geometric concepts of angles.

> The meaning of degree and radian measure of an angle.

> How to change from degrees to radians and vice versa.

> The definition of similar angles.

> How to find the sides of similar triangles using proportions.

> The Pythagorean Theorem and how to apply it.

> The trigonometric ratios of the right triangle: sine, cosine and tangent.

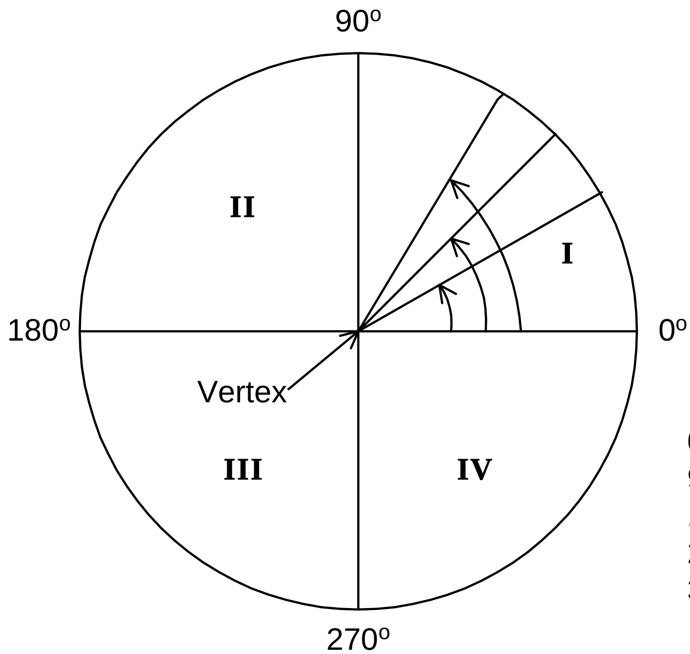
> How to solve triangles using the trigonometric ratios.

> How to use vectors and phasors.

3.1 Angles and Triangles

An *angle* is a measure of rotation between two radii in a circle.

The center of the circle where the radii meet is called the *vertex* of the angle.



0º = 0 rad
90º = Π/2 rad
180º = Π rad
270º = 3Π/2 rad
360º = 2Π rad

One complete rotation
= 360º = 2Π rad

A complete rotation (*counterclockwise*) is defined at 360^o.
It may also be defined as 2 Π radians, or rad.; therefore 360^o = 2 Π rad.
If 2Π rad $= 360^o$, then Π rad $= 180^o$, $\Pi/2$ rad $= 90^o$, etc.

$$1 rad = \frac{360}{2\Pi}$$

$$1 rad \approx 57.3^o$$

<u>*examples*</u>*:*

(1) change 2•/3 rad to degrees:

multiply by 180^o/• (57.3^o per rad)

$$\frac{2\pi}{3} x \frac{180}{\pi} = \frac{2x180}{3} = \frac{360}{3} = 120^{\circ}$$

(2) change 1.73 rad to degrees:

multiply by 180°/• (57.3° per radian)

$$1.73x\frac{180}{\pi} \approx 99^{\circ}$$

(3) change 135° to radians:

divide by 57.3°/radian:

$\frac{135}{57.3} = 2.36$ rad

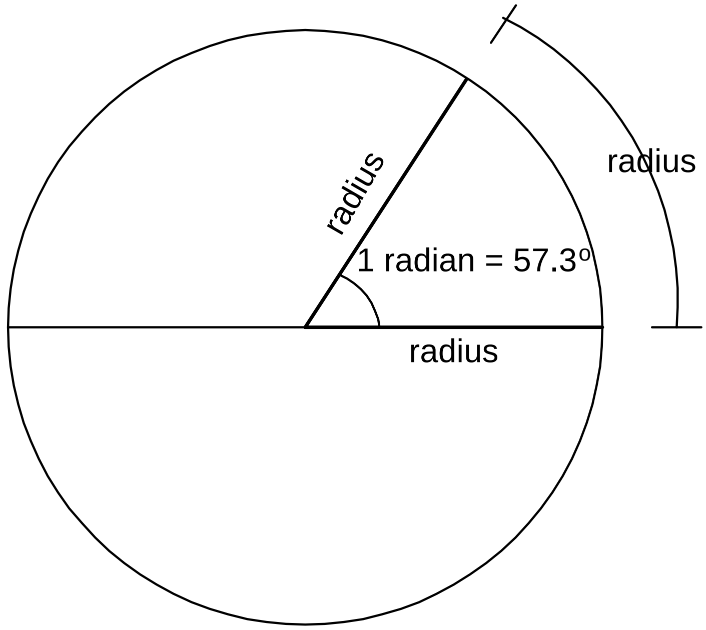
or multiply by $\frac{\pi}{180}$:

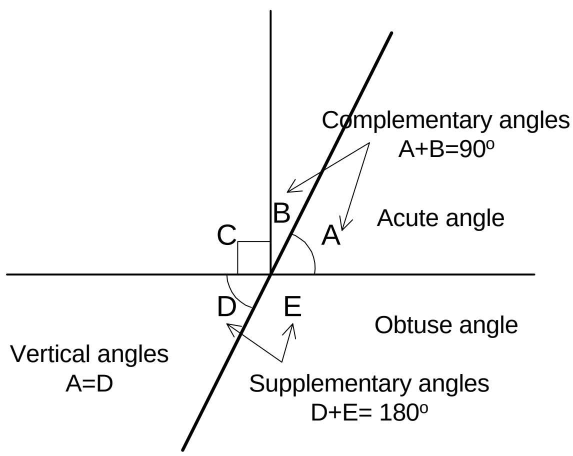
$135\frac{\pi}{180} = 2.36$ rad

(4) change 46° to radians:

$\frac{46}{57.3} = 0.8$ rad

The term “radian” is used because an angle of one radian cuts off an arc on the perimeter of the circle which is the length of one *radius*:





With respect to the above sketch:

A *right angle* is a 90^{o} angle. **C** is a right angle.

A *straight angle* is 180^{o}. The heavy line above is a straight angle.

An *acute angle* is greater than zero and less than 90^{o}. Angle **A.**

Perpendicular lines meet at right angles, as indicated by the square.

An *obtuse angle* is greater than 90^o and less than 180^o. Angle **E**.

Complementary angles are two angles that add up to 90^o. Angles **A** & **B**.

Supplementary angles are two angles that add up to 180^o. Angles **D** & **E**.

Vertical angles are opposite angles formed by two intersecting lines, and are equal. Angles **A** & **D**.

3.2 The Pythagorean Theorem

A *polygon* is a plane figure bounded by straight lines.

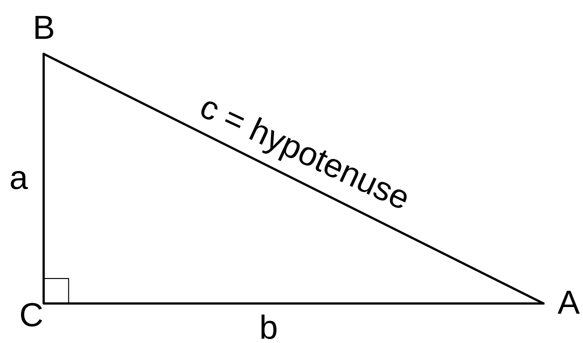
A *triangle* is a three-sided polygon.

A *rectangle* is a four-sided polygon.

A *pentagon* is a five-sided polygon, and so on with a *hexagon*, an *octagon* etc.

Every polygon can be divided into triangles, and any triangle can be divided into right triangles. The right triangle is, therefore, the basic building block for may figures.

The study of the right triangle is called *trigonometry*.



The longest side of any right triangle, which is opposite the right angle, is called the *hypotenuse*, labeled **c** above.

$A + B = 90^o$

The three angles of any triangle add up to 180^o.

Therefore angles **A** and **B** in a right triangle add up to 90^o and are complementary.

One of the most important and useful theorems in all geometry is the *Pythagorean theorem*. It was used by the Babylonian, Egyptian and Greek builders 3,000 years ago

The Pythagorean theorem states that the sum of the squares of the sides equals the square of the hypotenuse:

$$a^2 + b^2 = c^2$$

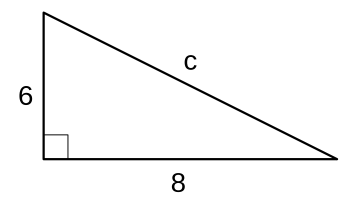
examples:

(5) if a = 6 and b = 8, what us the hypotenuse?

$a^2 + b^2 = c^2$

$c^2 = 6^2 + 8^2 = 36 + 64 = 100$

$c = \sqrt{100} = 10$



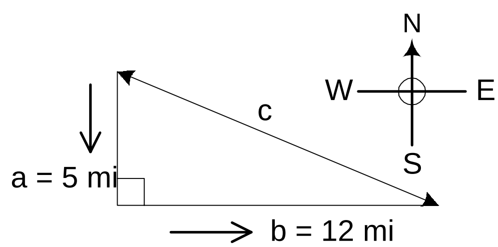
(6) A boat travels 5 mi. South and then turns and travels east for 12 mi. How far is it from the starting point?

$a^2 + b^2 = c^2$

$(5)^2 + (12)^2 = c^2$

$25 + 144 = c^2$

c = $\sqrt{25+144} = \sqrt{169} = 13$

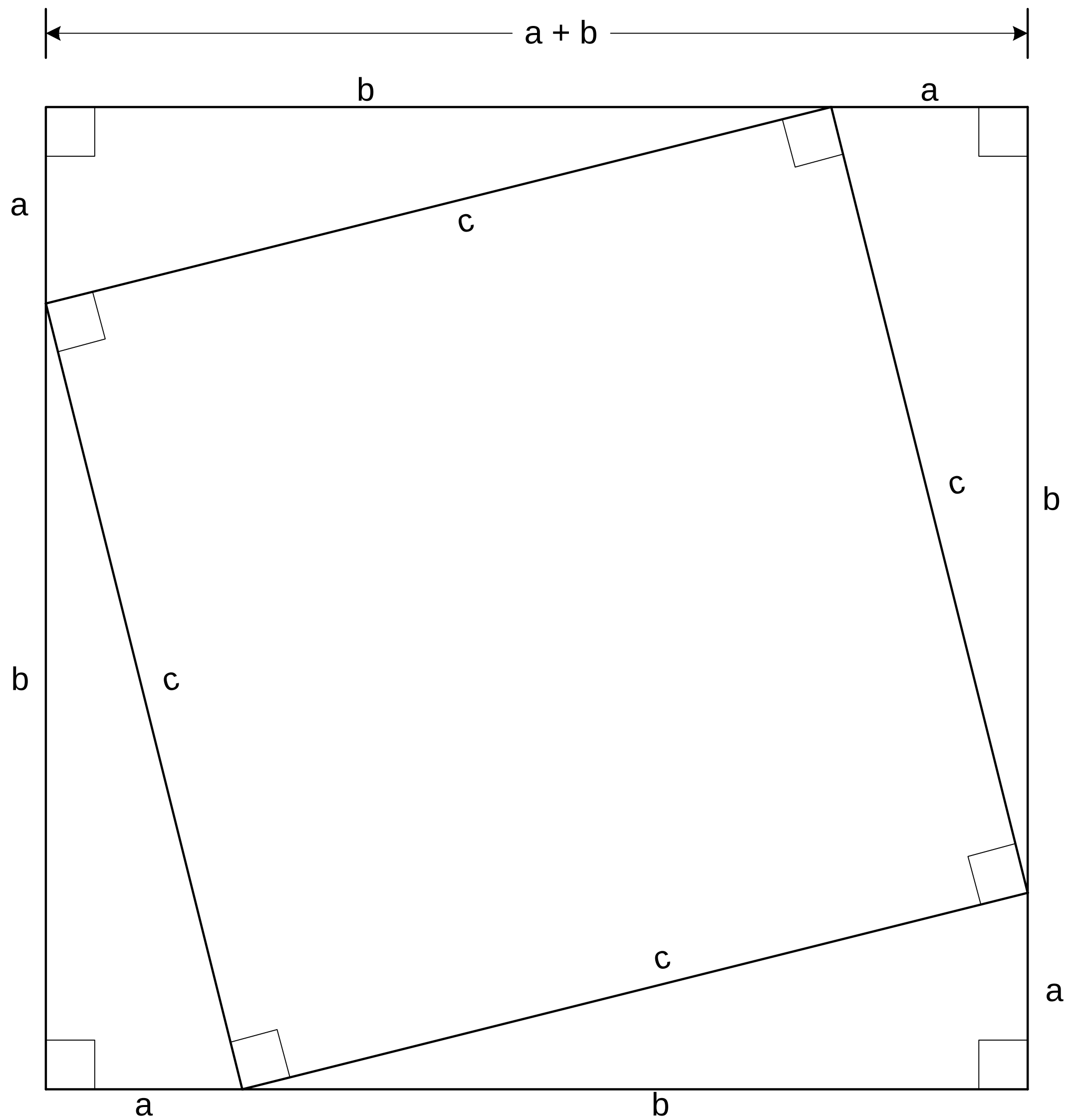


(note that $\sqrt{169} =$ either +13 or -13, but -13 has no physical significance in this example.)

3.3 *Proof of the Pythagorean Theorem*

If you are curious as to where this came from, the following is but one of many proofs.

Consider the following figure:



(Area of large square) = (Area of small square) + 4(Area of one right triangle)

$$(a+b)^2 = c^2 + 4\left(\frac{1}{2}ab\right)$$

$$a^2 + 2ab + b^2 = c^2 + 2ab$$

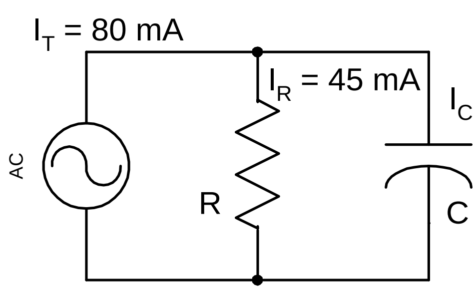
since the $2ab$ terms on both sides cancel:

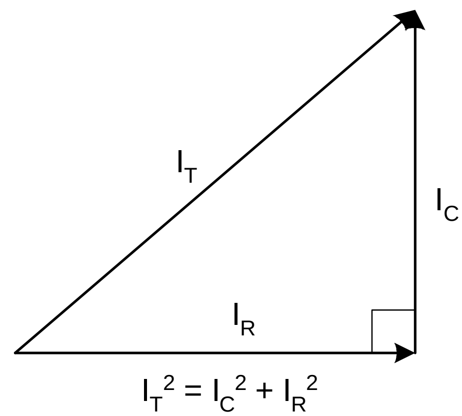
$$a^2 + b^2 = c^2$$

<u>examples</u>:

(7) Given the parallel RC electrical circuit below with an AC voltage source, we will later see that the current through the resistor and the capacitor are at right angles to one another, and that the total current supplied by the source is the hypotenuse of the resulting right triangle.

In this example the current through the capacitor is the unknown, which we may solve for using the Pythagorean Theorem.





Substituting the known currents:

$80^2 = 45^2 + I_C^2$ (where all units are in mA)

$6400 = 2025 + I_C^2$

$I_C^2 = 6400 - 2025 = 4375$

$I_C = \sqrt{4375} = 66$ mA

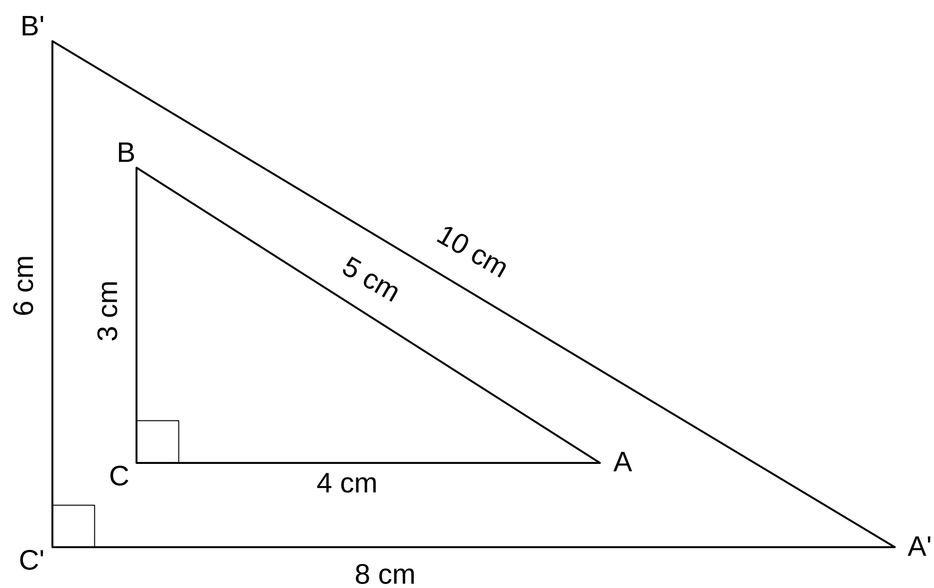
(again, only the positive root has meaning)

3.4 Similar Polygons and Triangles

An important relationship between polygons is *similarity*.

Similar polygons are two polygons that have the same shape but not necessarily the same size.

The *angles* of a polygon determine it's shape. Therefore, it all the angles of one polygon are equal to the corresponding angles of another polygon, the two are similar, as shown below.



Polygons, and particularly triangles, whose angles are the same and are similar have their *corresponding sides in proportion*.

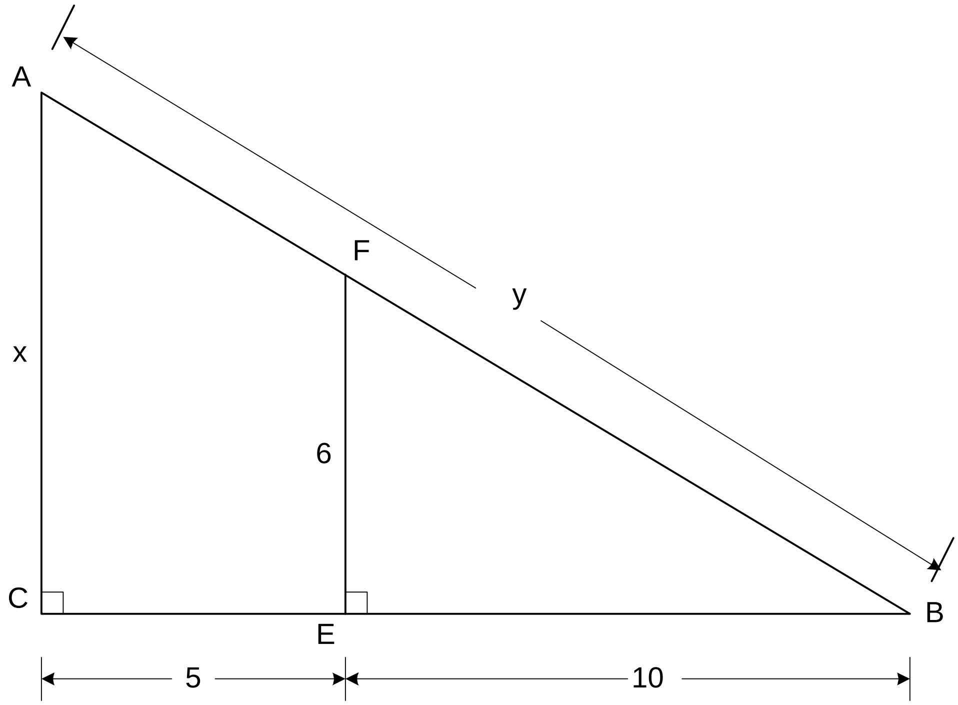
For the situation shown above:

$$\frac{10}{5} = \frac{8}{4} \qquad \frac{10}{5} = \frac{6}{3} \qquad \frac{8}{4} = \frac{6}{3}$$

<u>*examples*</u>*:*

(8) Given the similar right triangles shown below, where:

CE = 5, BE = 10 and FE = 6 Find side AC and hypotenuse AB



The two right triangles are similar because:

the right angles are equal and

the angle B is common to both triangles,

therefore the third angles, A and F, must be equal.

Since side AC corresponds to side FE, and side BC corresponds to side BE:

$$\frac{AC}{FE} = \frac{BC}{BE}$$

The figure shows that side AC = x, an unknown.

and: BC = 10 + 5 = 15

We also know that FE = 6, therefore we can rewrite the proportion:

$$\frac{x}{6} = \frac{15}{10}$$

by *cross multiplying*:

(x)(10) = (15)(6)

10x = 90

x = 9

You may find *y* by applying the Pythagorean Theorem, now that you have *x*:

$9^2 + 15^2 = y^2$

$y^2 = 81 + 225 = 306$

$y = \sqrt{306} = 17$

(9) A rectangular microprocessor chip is 3.3 mm x 0.4 mm. Wide. If the image of the chip under a microscope has a width of 4.2 cm, what is the length of the image and what is the scale of the image?

The microscope image is much larger than the actual chip. The sides of the image and the chip are proportional. Let (*x*) = length of the image. To find (*x*), write the image-to-object proportion:

$$\frac{x}{0.4mm} = \frac{4.2cm}{3.3mm}$$

Note the units. If the units for the numerators agree and the units for the denominators agree, then the ratios will be equal.

Cross multiply to obtain the solution:

$$(x)(3.3mm) = (4.2mm)(0.4cm)$$

$$x = \frac{(4.2mm)(0.4cm)}{3.3mm} = 5.1cm$$

Note that the (mm) terms cancel, leaving (cm) units for the answer.

To have converted all dimensions to (mm) units would have been a safer and trouble-free procedure.

The question also asked for the scale of the image:

$$scale = \frac{9.4cm}{3.3mm} = \frac{940mm}{3.3mm} = \frac{285}{1}$$

<u>Practice Problems</u>:

Kramer, pages 305 through 308, questions 1 -46.

3.5 Trigonometry of the Right Triangle

Two or more right triangles that have the same angles are similar. Therefore, they have the same ratios for any two corresponding sides.

These ratios are very useful for finding sides and angles of right triangles when you know only some of the sides or angles.

They are called trigonometric ratios, or trigonometric functions.

Given the right triangle below, the three basic trigonometric functions for the acute angle (A) are:

$$\sin eAor \sin A = \frac{opposite}{hypotenuse} = \frac{a}{c}$$

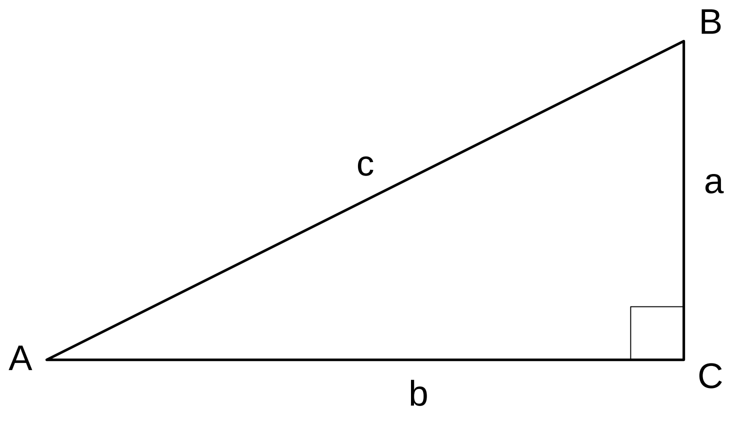
$$\cos ineAor \cos A = \frac{adjacent}{hypotenuse} = \frac{b}{c}$$

$$\tan gentAor \tan A = \frac{opposite}{adjacent} = \frac{a}{b}$$

A mnemonic for remembering these definitions is:

"S-OH, C-AH, T-0A"

(sin-Opp/Hyp, cos-Adj/Hyp, tan-Opp/Adj)



sin A = cos B

cos A = sin B

examples:

(10) Given he triangle above, with:

a = 9 and b = 12,

find the values of the sine, cosine and tangent of angles (A) and (B) using the formulas above.

First, calculate the hypotenuse (c) using the Pythagorean Theorem:

$c^2 = a^2 + b^2$

$c^2 = 9^2 + 12^2$

$c^2 = 81 + 144 = 225$

$$c = \sqrt{225} = 15$$

Then apply the definitions from the prior page:

$$\sin A = \frac{a}{c} = \frac{9}{15} = 0.60 \qquad \sin B = \frac{b}{c} = \frac{12}{15} = 0.80$$

$$\cos A = \frac{b}{c} = \frac{12}{15} = 0.80 \qquad \cos B = \frac{a}{c} = \frac{9}{15} = 0.60$$

$$\tan A = \frac{a}{b} = \frac{9}{12} = 0.75 \qquad \tan B = \frac{b}{a} = \frac{12}{9} = 1.33$$

(11) Using a calculator, find the following:

a) sin 50.3 degrees

b) tan 1.20 rad

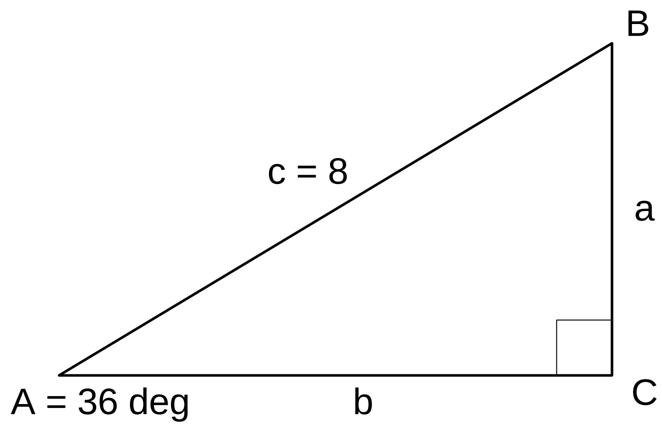
c) $\cos\frac{\pi}{5} rad$

(12) Given a right triangle with:

A = 36 degrees

side (c) = 8

Solve for the missing parts of the triangle.



You need to find:

angle (B) and sides (a) and (b)

A + B = 90 deg

B = 90 - A = 90 - 36 = 54 deg

$$\sin A = \frac{a}{c}$$

$$\sin 36° = \frac{a}{8}$$

$$a = (8)(587) = 4.70$$

$$\cos A = \frac{b}{c}$$

$$\cos 36° = \frac{b}{8}$$

$$b = (8)(0.809) = 6.47$$

(13) Given a right triangle with:

a = 6.2 and b = 5.5

Find the angle (A).

$$\tan A = \frac{a}{b} = \frac{6.2}{5.5} = 1.13$$

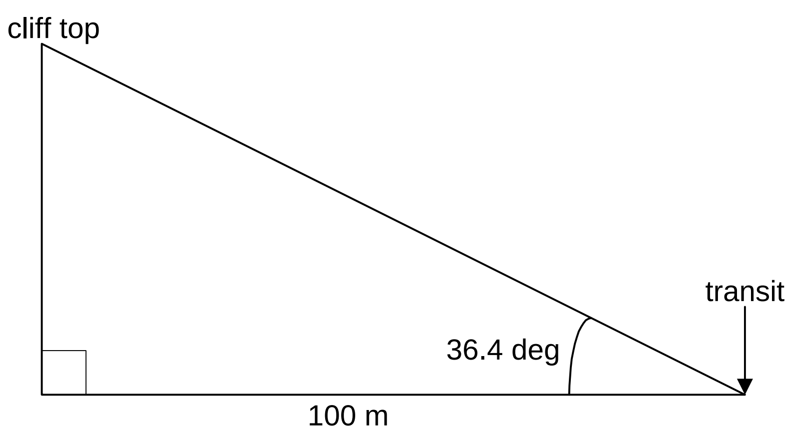
Now you need to know what angle has a tangent of 1.13.
This is known as an *inverse* trigonometric function, written:

$\tan^{-1} 1.13$

On the calculator, enter 1.13 followed by **2nd** function and TAN.
The result is 48.5 deg.

(14) To find the height of a cliff, the surveyor chooses a point 100 m from the base of the cliff and sets up his transit. At this point he measures an elevation angle of 36.4 deg to the top of the cliff.

What is the height (h) of the cliff (ignoring the fact that the transit is not quite at ground level, a correction for which could be made later)?



$$\tan 36.4° = \frac{opposite}{adjacent} = \frac{h}{100}$$

$$h = 100(\tan 36.4)$$
$$h = 100(0.737) = 73.7m = 74m$$

(Subtracting 5 to 6 feet, as appropriate, would compensate for the fact that the transit is at eye level, not ground level.)

<u>Practice Problems</u>:

Kramer, pages 313 to 315, problems 1 - 50.

3.6 Trigonometry of the Circle

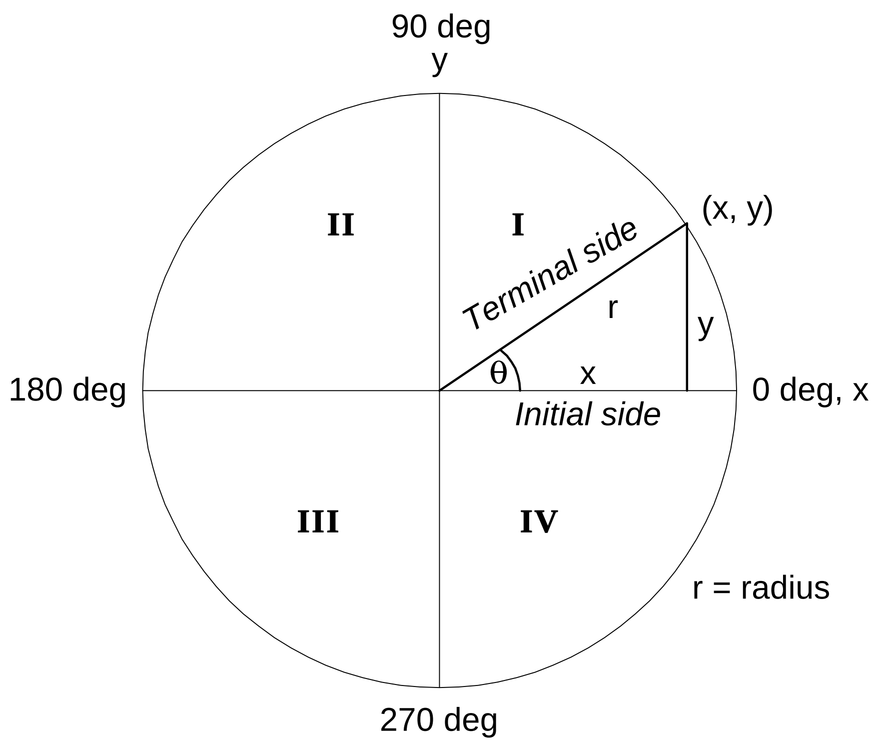
Since angles measure rotation, or circular motion, we can define an angle, positive or negative, in a circle.

In the following circle the center is the origin of a rectangular coordinate system.

The direction of the radius is shown by the angle θ (the Greek letter *theta*).

Counterclockwise rotation is considered *positive*, and clockwise rotation *negative*.

The circle contains four *quadrants*, lettered with Roman Numerals **I** through **IV**.

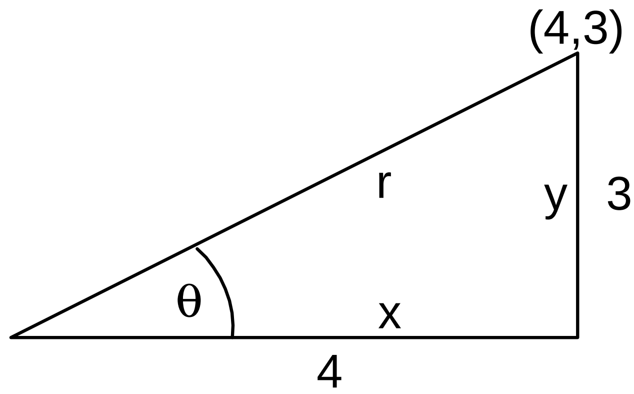


The three trigonometric functions for the angle θ whose terminal side passes through a point (x, y) on a circle of radius r are:

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

examples:

(15) Draw the triangle and find the three trigonometric functions for the angle θ_1 (4,3):



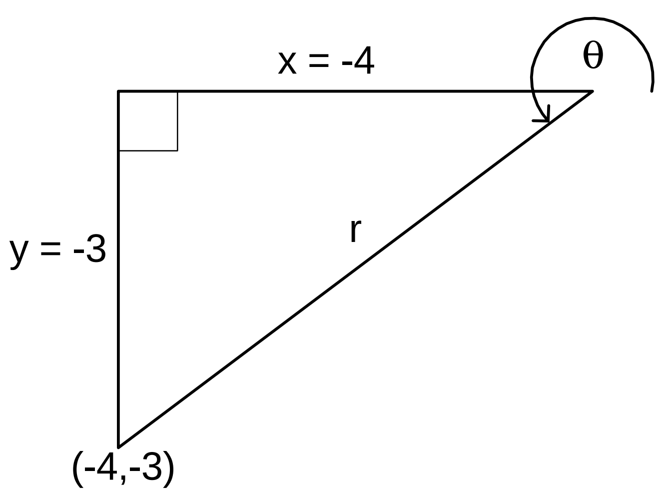
The radius, r, is found from the Pythagorean Theorem:

$$r^2 = x^2 + y^2$$

$$r = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$\sin\theta_1 = \frac{3}{5} \qquad \cos\theta_1 = \frac{4}{5} \qquad \tan\theta_1 = \frac{3}{4}$$

(16) Draw the triangle and find the three trigonometric functions for the angle θ_3 (–4,–3):

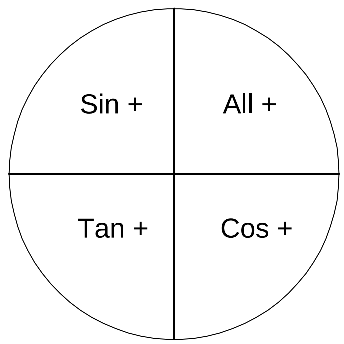


$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$\sin\theta_3 = \frac{-3}{5} = -0.6 \qquad \cos\theta_3 = \frac{-4}{5} = -0.8 \qquad \tan\theta_3 = \frac{-3}{-4} = \frac{3}{4} = 0.75$$

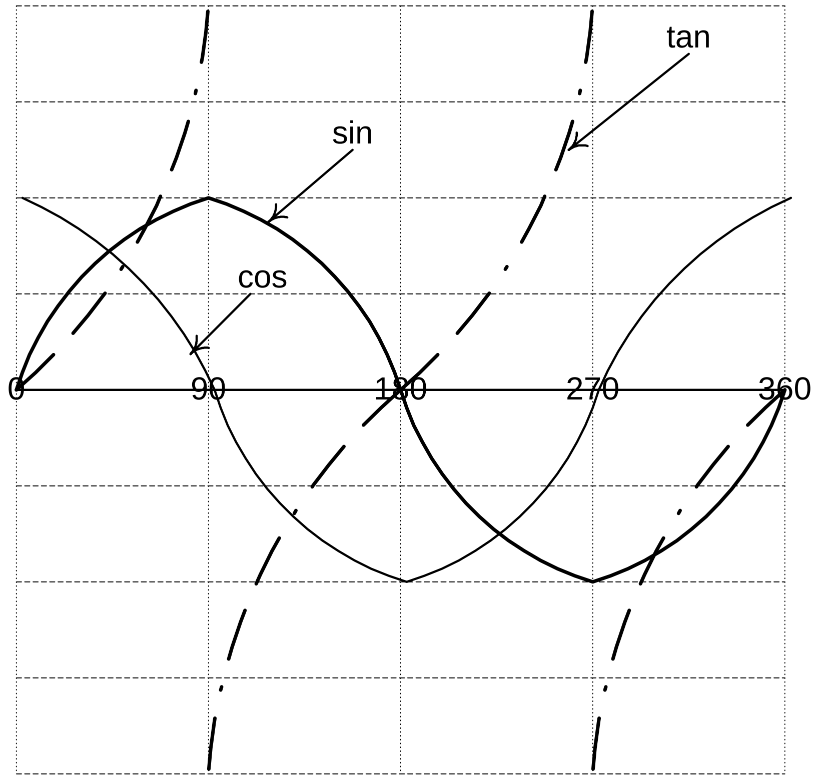
All the functions are positive in the first quadrant. You will find that only one function is positive in each of the other quadrants.



"All Star Trig Class"

Rather than remember the above, it might be easier (and also useful for other applications) to have a mental picture of the nature of the sin, cos and tan curves, all plotted together.

The sin and cos are easy to remember; we don't work with the tan curve as frequently.



Practice Problems:

Kramer, pages 323 - 325, problems 1 - 52.

3.7 Inverse Functions

A prior example (13) introduced the *inverse* tangent function which is used to *find the angle* when the *value of the tangent is known*.

It's just a matter of going backwards!

examples:

(17) Given cos θ = -0.250, find the angle θ.

With the convenience of a calculator:

1) Enter 0.250 followed by the change sign key for –0.25..

2) Note that *above* the **cos** key is written **cos^{-1}**, which is accessed by first pressing the **2nd** key.

So press **2nd** followed by **cos** and read 104.5°

3) Look at the sin, cos & tan curves above and note that cos is negative in the second and third quadrants; therefore 104.5 deg is a valid answer.

360 - 104.5 = 255.5 deg is also valid.

4) The calculator will always give you the first quadrant angle. This is called the *principal* value.

(18) Find tan^{-1} -0.6783.

1) Enter -0.6783.

2) Press **2nd** and **tan** to read -34.15°

Practice Problems:

Kramer, pages 329 - 330, problems 1 - 34.

3.7 The Concept of Vectors and Phasors

Vectors and phasors are quantities, which need *two numbers* to describe them.

Force is a vector quantity, as is AC voltage. Either can be represented on a graph as an arrow starting at the origin and ending at a point (x,y).

The length of the arrow indicates *magnitude*; the angle indicates it's *direction*.

The difference between a vector and a phasor is in the measurement of the direction.

> A vector's direction is an angle in two or three-dimensional space.
>
> A phasor's direction is based on time. The angle of a phasor represents a time difference between two quantities, such as voltage and current in a circuit.

Examples of vectors and phasors are:

> *Velocity vector:* The velocity of the wind is 20 mi./h from the SE.
>
> *Electromagnetic force vector:* A force exerted by an electromagnetic field on a particle is 2 x 10^{-6} N at an angle of 30 deg. with the horizontal.
>
> *Voltage phasor:* the voltage in an RL series circuit is 120 V with a phase angle of 47 deg. this is written in polar form as: $120\angle 47° V$
>
> *Current phasor:* The total current in an RL parallel circuit is 50 mA with a phase angle of -15 deg. This is written in polar form as: $50\angle -15° mA$

Most quantities we are familiar with can be described with <u>one number</u>. These are called *scalars*. Height, weight and distance are examples of scalars.

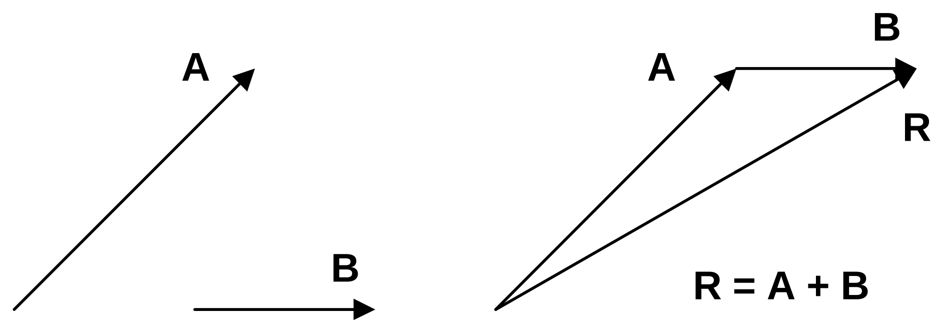
3.8 Adding Vectors and Phasors

Vectors and phasors are drawn as arrows and are usually identified with capital or bold letters.

the length of the arrow represents the magnitude,

the arrowhead shows the direction.

The following shows how to <u>add</u> two vectors or phasors graphically.



The tail of **B** is placed at the head of **A** without changing it’s direction.

The resultant *(sum)* goes from the tail of **A** to the had of **B**.

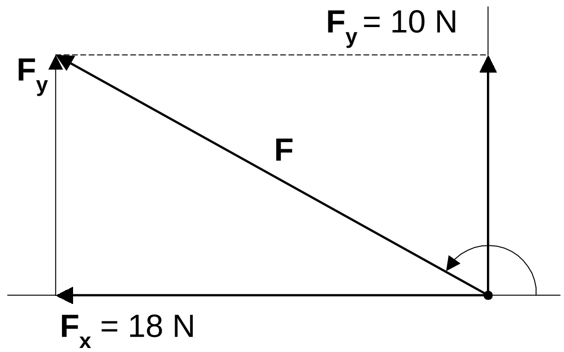
<u>examples</u>:

(19) In this figure a body is acted on by two forces:

a horizontal force $\mathbf{F_x}$ = 18 N at an angle of 180°

a vertical force $\mathbf{F_y}$ = 10 N at an angle of 90°

Find the resultant force **F** acting on the body.



Since the force is a vector, you need to find its magnitude *and* direction.

Find the magnitude by applying the Pythagorean Theorem:

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{18^2 + 10^2} = \sqrt{324 + 100} = \sqrt{424} = 20.6N \approx 21N$$

To find the direction, you need the angle in the second quadrant:

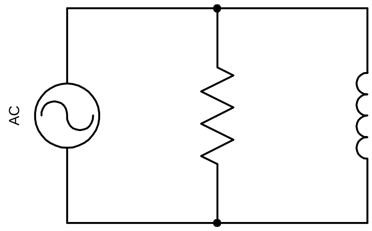
$$angle = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{10}{18}\right) = 29.1\deg$$

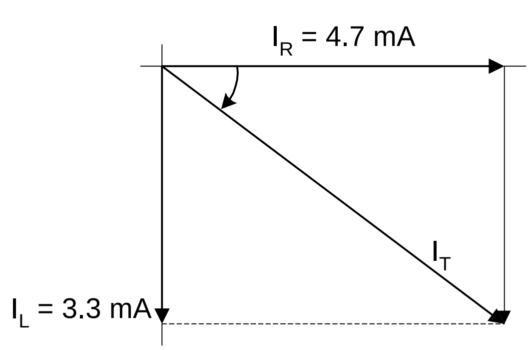
but this is the angle between the **F** and $\mathbf{F_x}$ vectors.

the angle we want is given by:

180 - 29.1 = 150.9 = 151°

(20) The following sketch presents a phasor diagram for the currents in an RL circuit. In such a circuit, the current in the inductor always *lags* the current in the resistor by 90°.





Note the negative angle, which indicates that I_L lags I_R by 90 degrees.

Find the magnitude of I_L using the Pythagorean Theorem:

$$I_T = \sqrt{I_R{}^2 + I_L{}^2} = \sqrt{4.7^2 + 3.3^2} = \sqrt{22.09 + 10.89} = \sqrt{32.98} = 5.7mA$$

$$angle = \tan^{-1}\left(\frac{-I_L}{I_R}\right) = \tan^{-1}\left(\frac{-3.3}{4.7}\right) = -35.1 \approx 35\deg$$

Written in polar form, the solution is:

$I_T = 5.7\angle -35^\circ$ ma

<u>Practice Problems</u>:

Kramer, pages 335 - 337, problems 1 - 30.

There are also 40 good review problems on pages 345 & 346.

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Chapter 4

Introduction to Calculus

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Introduction to Calculus

This brief Section seeks only to provide the reader with a very brief and general concept of what calculus is all about.

The study of calculus is customarily divided into two parts:

Differential calculus, and,

Integral calculus.

4.1 Differential Calculus

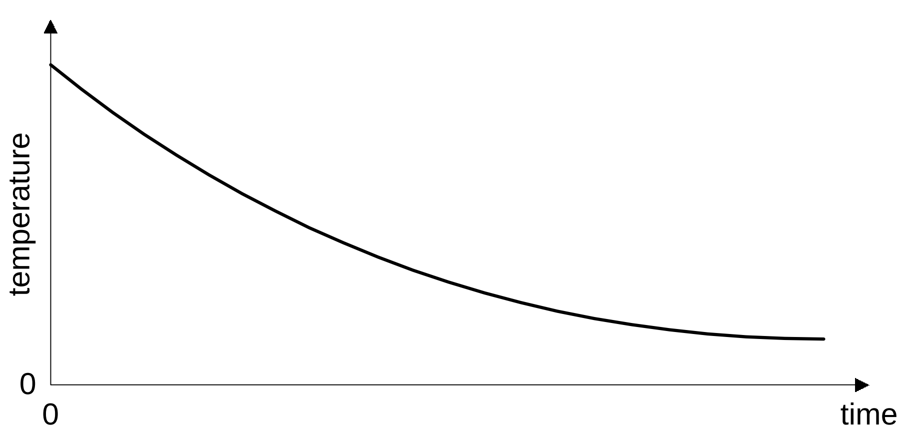
Differential calculus is concerned with the ***rate of change*** of one variable with respect to another.

Differential calculus is exemplified by the following questions:

What is the best way of describing the speed of a car or the cooling of a hot object?

How does the change of output current of a transistor amplifier circuit depend upon the change of the input current?

Consider the following curve of the temperature of a hot object:



These curves shows that the temperature of the hot object is decreasing most rapidly at *t = 0*. As time increases, the temperature of the hot object decreases more slowly until it reaches an equilibrium with its surrounding.

This is as you would expect, since the cooling of a hot object is dependent upon the temperature *difference* between it and its surrounding.

If we define *h* as the heat transfer, at *t = 0* we could say:

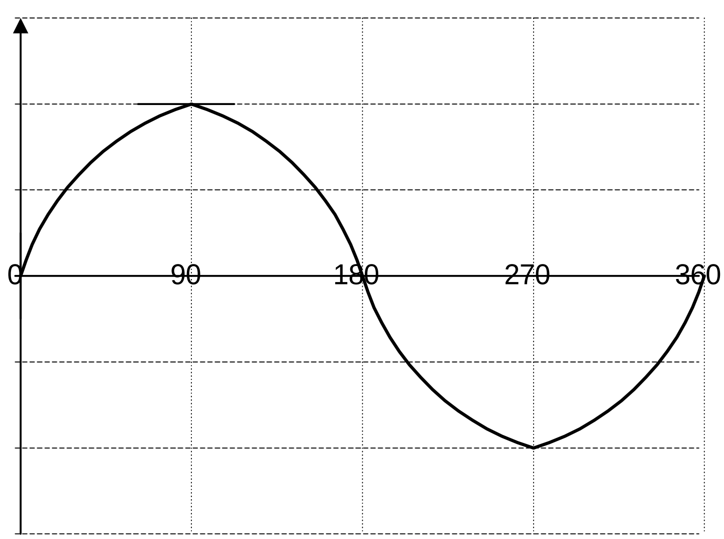
$$\frac{\Delta h}{\Delta t} = \frac{dh}{dt} \text{ is maximum at } t = 0$$

At *t* increases, we have noted that *h* decreases, hence we could say that

$$\frac{\Delta h}{\Delta t} = \frac{dh}{dt} \text{ is decreasing as time increases}$$

Note that the *slope* of the *h vs. t* curve is <u>greatest</u> for maximum heat transfer, <u>decreasing to zero</u> when the hot object reaches thermal equilibrium with its surrounding.

As another example which is important to electricity, consider the following sine wave plot of voltage, *v vs. time:*



The slope of the sine wave curve is at a maximum at 0, 180 and 360 degrees. Precisely at these points the sine curve is vertical (the tangent is a vertical line), which is maximum slope.

Which is the same as to say:

$$\frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \max \text{ at 0, 180, 360, etc.}$$

And at 90, 270 degrees, etc., the slope of the curve is zero.
Precisely at these points the curve is horizontal, it is reversing (the tangent is a horizontal line), which is minimum slope.

Which is the same as to say:

$$\frac{\Delta v}{\Delta t} = \frac{dv}{dt} = 0 \text{ at 90, 270, etc.}$$

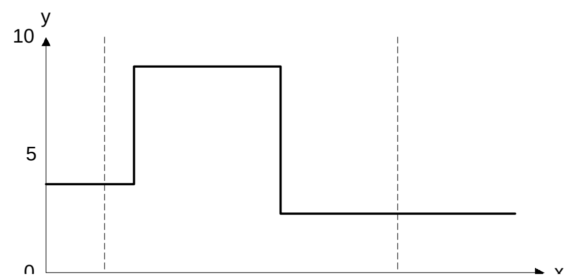
Thus we see that the sine wave alternates between a *maximum rate of change* and a *minimum rate of change* every 90 degrees (or quarter cycle).

4.2 Integral Calculus

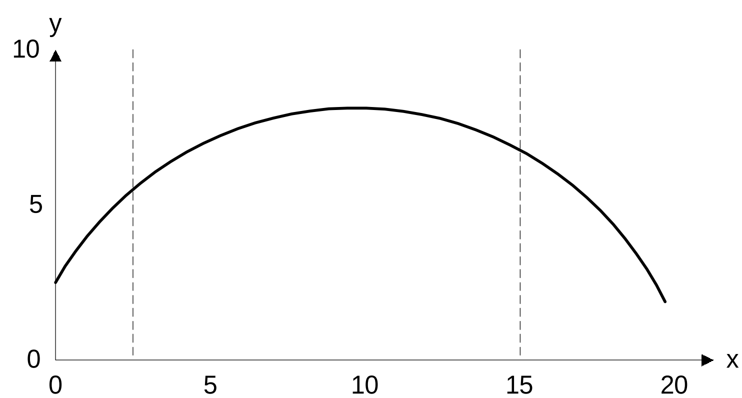
Integral calculus is largely concerned with *areas*.

If you were given the following curve and asked to calculate the area under the curve between the marked points: *t = 3* and *t = 15* you would have no difficulty.

You would compute the area by summing three separate smaller areas.



However, if the curve is irregular, like the following, you face a much more difficult problem:



Integral calculus approaches this area problem by slicing such a curve into an infinite number of thin vertical slices, each of width approaching zero, and summing them.

If we can define the shape of the curve by an equation, finding the area between *t = 3* and *t = 15* becomes rather simple.

So now if you see an expressing such as:

$$\sum_{x=15}^{x=3} f(x)dx$$

you will have the general idea of what it is used for.

How to evaluate such an equation is beyond our needs and hence “beyond the scope of this course.”

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Chapter 5

Introduction to Logarithms

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Logarithms

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Learning Objectives

In this Chapter you will learn:

> The definition of an exponential function.

> The definition of a common logarithm.

> How to graph exponential growth and decay functions.

> The definition of a logarithm.

> The meaning of the exponential number *e.*

> How to solve exponential equations.

> How to compute power, current and voltage gain in typical electrical situations.

5.1 What is a Logarithm?

A logarithm is an exponent.

Logarithm is simply written, ***“log”***

Logarithms simplify calculations, and allow us to solve equations with exponents. Consider the following discussion:

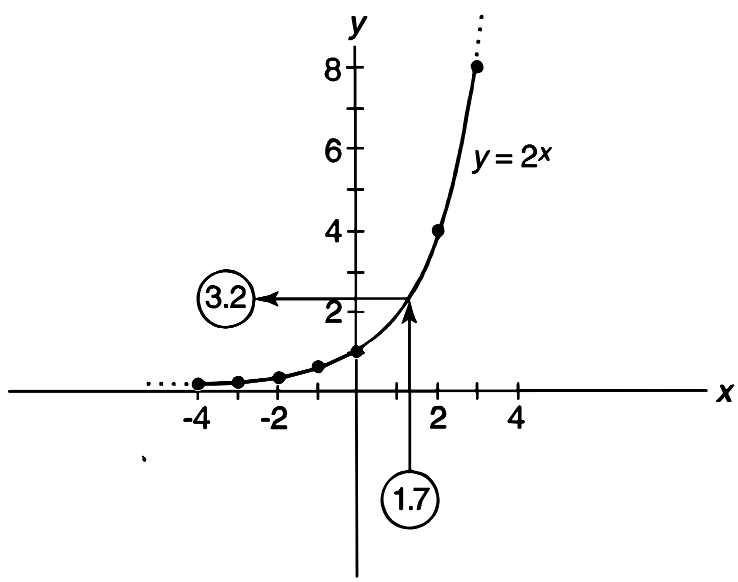
5.2 Well, what are Exponential Functions?

An *exponential function* is defined as:

$$y = b^x$$

The following figure shows a specific exponential function:

$y = 2^x$ which plots as:



Note that as the value of ***x*** increases, ***y*** increases at a faster and faster rate, and becomes very large. The plot gets steeper and steeper, and is said to *increases exponentially*.

When $x = 1.7$, the plot shows that $y = 3.2$
(Which can be more exactly solved on the calculator.)

Such a plot is called an *exponential growth function*. Many natural growth patterns behave in this way.

> Plants and animals, in their early stages, grow exponentially,

> Animal populations tend to grow exponentially, when uncontrolled,

> The amount of money in a savings account grows exponentially at a constant compound interest rate.

Also note that as $\boldsymbol{x}$ decreases, $\boldsymbol{y}$ decreases at a slower and slower rate, and becomes infinitely small. The curve approaches the horizontal $\boldsymbol{x}$ axis as $\boldsymbol{y}$ gets closer and closer to zero.

The $\boldsymbol{x}$ axis is called a *asymptote* of the graph. However $\boldsymbol{y}$ never attains zero, and never becomes negative. This decrease in $\boldsymbol{y}$ is the opposite of exponential growth, and is called *exponential decay*.

5.3 Common Logarithms:

Common logarithms use the number 10 as their base. You have gained experience working with "powers of 10" in sections 1.4 through 1.8 of this booklet.

A common logarithm is the exponent (or power) to which you must raise 10 to get a certain number.

This is simply written, ***"log_{10}"*** or ***"log"*** (base 10 understood).

examples are:

(1) $10^3 = 1{,}000$
the log of 1,000 is, therefore, 3

(2) $10^6 = 1{,}000{,}000$
the log of 1,000,000 is, therefore, 6

Which is also to say:

if: $N = 10^x$

then: $\log(N) = x$

Sometimes you will see this written as:

$$\log_{10}(N) = x$$

which ensures that you know the base is 10

Logs for numbers smaller than 10 are *less than 1*. For numbers larger than 10 the log is *greater than 1*.

From the definition of a log, we can write:

$$2 = 10^{x}$$
$$\log 2 = \log\left(10^{x}\right)$$
$$\log 2 = x$$

For which the easiest way to find the value of $\boldsymbol{x}$ is from your calculator:

> Enter the number whose log you want to find,

> Press the button labeled LOG

> Which displays 0.301 as the value of $\boldsymbol{x}$.

Using your calculator, find that: $\log(1) = 0$
Because *anything* (including 10), raised to the zero power, is 1.

The **inverse log** is called the *antilog*, often written $\log^{-1}$.
(This is the same as inverse trig functions.)

When we know the log and want to find the original number, we want the antilog.

To find the antilog, simply raise 10 to the given power.
(Use your calculator *2nd* & *10X* keys for this.)

for example:

(3) what is the antilog of 1.845?
$10^{1.845} \cong 70$

5.4 Logarithms to the Base e:

The second base that is frequently use for logarithms is usually represented by ***e***.

$$e \cong 2.718 \quad \text{(an } \textit{irrational number}\text{)}$$

Logarithms that use ***e*** for their base are called *natural logarithms*, and are written as:

$$\log_e \quad \text{or} \quad \ln$$

Your calculator has keys for *LN* & e^x just to the right of the comparable keys for *LOG* & 10^x.

for example:

(4) $\ln 2 = 0.693$

(5) $\ln 6 = 1.792$

(6) $\ln 32 = 3.466$

Computers often work only with natural logarithms. Conversion between the two types of logarithms is easily done.

5.5 Decibels:

The ***bel*** (abbreviated *B*) is named after Alexander Graham Bell, who did pioneering work with sound and the way our ears respond to it.

Our ears respond to sounds ranging from an intensity of less than 10^{-16} W/cm^2 to intensities larger than 10^{-4} W/cm^2 (where we begin to experience pain). This is a range of more than 10^{12} times from the softest to the loudest sounds.

Logarithms provide a convenient way to represent these values, because they compress this scale into a range of <u>12</u>, rather than a range of <u>one trillion</u>.

The log functions have the result of compressing numbers. Numbers between

$$\begin{aligned} 1 - 10 &\Rightarrow 0 - 1 \\ 10 - 100 &\Rightarrow 1 - 2 \\ 100 - 1{,}000 &\Rightarrow 2 - 3 \\ 1{,}000 - 10{,}000 &\Rightarrow 3 - 4 \end{aligned}$$

A *bel* is defined as the *logarithm of a power ratio*. It gives us a way to compare power levels with each other, and with some reference power level.

$$bel = \log\left(\frac{P_1}{P_0}\right)$$

Where $\boldsymbol{P_o}$ is the reference power, or the power you want to use for comparison, and $\boldsymbol{P_1}$ is the power you are comparing to the reference level.

We also use this to compare electrical power levels.

The *decibel* is one-tenth of a bel, abbreviated $\boldsymbol{dB}$.

Written directly in $\boldsymbol{dB}$, the above becomes:

$$dB = 10\log\left(\frac{P_1}{P_0}\right)$$

for example:

(7) How many $\boldsymbol{dB}$ does the power increase if an amplifier takes a 1 W input signal, and boosts it to 50 W?

$$dB = 10\log\left(\frac{50}{1}\right) = 10\log(50) = 10(1.699) = 16.99$$

Sometimes when we are comparing signal levels in an electronic circuit, we know the voltage or the current, but not the power. We can calculate the power, if we know the impedance level.

We can take a shortcut to comparing the signal levels in $\boldsymbol{dB}$ as long as the impedance is he same in both circuits. From Ohm's Law and the power equation we know that:

$$P = \frac{E^2}{R} \qquad \text{and} \qquad P = I^2 R$$

So we can use E^2 or I^2 in place of power:

$$dB = 10\log\left(\frac{E_1}{E_0}\right)^2 = 20\log\left(\frac{E_1}{E_0}\right)$$

and

$$dB = 10\log\left(\frac{I_1}{I_0}\right)^2 = 20\log\left(\frac{I_1}{I_0}\right)$$

There are a few power ratios worth remembering.

examples are:

(8) Lets look at doubling the power of an amplifier. It doesn't matter if we are going from 1 W to 2 or from 500 W to 1,000. The ratio is still 2 in doubling the power level.

To find the $\boldsymbol{dB}$ increase:

$$dB = 10\log(2) = 10(0.301) = 3.01 \cong 3$$

Thus, anytime you *double* the power represents a *3 dB increase*.

(9) Cutting the power in half becomes:

$$dB = 10\log(0.5) = 10(-0.301) = -3.01 \cong -3$$

Anytime you *halve* the power represents *-3 dB,* a *decrease*.

It is often convenient to compare a certain power level with some standard reference.

Suppose you measured the signal coming into a receiver from an antenna and found the power to be 2 x 10^{-13} mW. As this signal goes through the receiver it increases in strength until it finally produces sound in the speaker. It is convenient to describe these signal levels in decibels.

A common reference level is 1 mW. The decibel value of the signal compared to 1 mW is referred to as ***"dBm,"*** which mean decibels compared to 1 mW into a 600 Ω resistive load.

In our example, the signal strength at the receiver input is:

$$dBm = 10\log\left(\frac{2x10^{-13}}{1}\right) = 10\log\left(2x10^{-13}\right) = 10x(-12.7) = -127dBm$$

Many other reference powers are used, depending upon the circuits and power levels.

If 1 W is used as the reference, it is specified as $\boldsymbol{dbW}$.

Antenna power gains are often specified in relation to a dipole ($\boldsymbol{dBd}$) or an isotropic radiator ($\boldsymbol{dBi}$).

Any letter following $\boldsymbol{dB}$ lets you know that some reference power is being specified.

If two quantities are compared in relation to a process (amplification, attenuation, etc.), and results are stated in $\boldsymbol{dB}$, generally the equation

$$20\log\left(\frac{output}{input}\right)$$

was used to make the computation.